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## Theoretical Computer Science

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## Preface

Modern computations in the sciences, engineering, and signal and image processing rely on numerical and symbolic (algebraic) algorithms. Numerical algorithms save computer memory and running time by confining the computations to finite precision and allowing rounding errors. They require special care in order to avoid error propagation and magnification, and to validate the output. Symbolic (algebraic) algorithms, on the other hand perform computations with no errors by using more computer time and memory.

For many computational tasks, algorithms of both types can be applied, and frequently the best performance is achieved in *hybrid algorithms*, which combine the power of symbolic and numerical techniques. Quite typically this is the case in computations with polynomials and general and structured matrices, for example, in root finding for univariate polynomials and systems of multivariate polynomials, computing approximate greatest common divisor of polynomials, polynomial factorization and optimization, polynomial and rational interpolation, solving linear systems of equations, and computation of matrix inverses, determinants and characteristic polynomials. The symbolic-numerical benefits and opportunities have been studied since the early 1990s with growing intensity and recognition (see, e.g., [1–10]).

This special issue presents papers on numerical and symbolic algorithms by following the TCS tradition established in 2004 (see [11,12]). Staying within the traditional scope of the fields, some papers in the issue bring radically new insights into the subjects.

Annie Cuyt and Wen-shin Lee explore the classical problem of rational interpolation, but introduce novel techniques. The arithmetic time of their solution algorithm depends only on the number of monomials in the numerator and denominator rather than on the degree  $d$  and the number of variables  $n$ . This enables a dramatic acceleration of the known algorithms in the most realistic case of sparse inputs where the standard methods run in time of the order  $d^n$ , exponential in  $n$ . The algorithms have been devised for both numerical and symbolic implementations.

Philippe Pébay, J. Maurice Rojas and David C. Thompson take a similar point of view in polynomial optimization and accelerate the known algorithms from exponential to subquadratic time, although in the rather special case of an  $(n+2)$ -nomial.

Gudmund Skovbjerg Frandsen and Piotr Sankowski propose a new approach to the classical problem of the evaluation of the characteristic polynomial of a matrix. Their work has further applications to some fundamental matrix computations, for example, to computing the matrix determinants, rank and spectrum. The authors employ the Lanczos algorithm and the reduction to the Frobenius normal form, to direct their algorithms to fast dynamic updating of the output values as the input matrix is updated by means of its rank-one modifications. Under this realistic assumption, they dramatically improve the known algorithms.

Michail Kourmiotis, Marilena Mitrouli and Dimitrios Triantafyllou present a new parallel algorithm for computing an approximate greatest common divisor of a univariate polynomial, which is among the most studied topics of symbolic-numerical computing. Their main technical novelty is the structure-preserving rearrangement of the associated generalized Sylvester matrix, which enables dramatic parallel acceleration of the known algorithms in the case where the input consists of many polynomials. Their formal estimates are supported by the results of extensive experimental computations.

Two papers, one by André Galligo and Adrien Poteaux and the other by Carlos Hoppen, Virginia Rodrigues, and Vilmar Trevisan are devoted to bivariate polynomial factorization, which is another fundamental subject of symbolic-numerical computing.

The former paper improves the known algorithms in the special but still interesting case where the input is the product of polynomials with random coefficients of a limited size. The paper demonstrates the power of the important general techniques of homotopy continuation and randomization.

The latter paper extends Gao's factorization algorithm to the field of small positive characteristics and derandomizes the computations.

Sylvain Chevillard, John Harrison, Mioara Joldes and Christoph Lauter study automated validation of numerical polynomial approximation of real functions (including the technically harder case of functions with removable discontinuities). The goal is to bound the absolute or relative error between a real function and the approximating polynomial. The authors carefully study the relevance to the issue of a number of techniques from interval arithmetic, symbolic computation and automated proof checking, both in theory and practice. They propose a specific algorithm based on these techniques towards obtaining a formal proof of the bound. In their implementation, part of the proof is already verified using a formal proof checker.

All papers in this special issue have been refereed according to TCS standards; acceptance required the consent of at least two reviewers. The acceptance rate was 1/3. Some papers came from the SNC 2009 Workshop on Symbolic-Numerical Computations (SNC 2009) in Kyoto, Japan, in 2009, but some other authors submitted the journal versions of their proceedings papers to the regular issues of the TCS or CAMWA. These two journals maintain interest in publication of papers on symbolic and numerical computations.

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## References

- [1] V.Y. Pan, Complexity of computations with matrices and polynomials, *SIAM Review* 34 (2) (1992) 225–262.
- [2] D. Bini, V.Y. Pan, *Polynomial and Matrix Computations, Volume 1: Fundamental Algorithms*, Birkhäuser, Boston, 1994.
- [3] V.Y. Pan, Solving a polynomial equation: some history and recent progress, *SIAM Review* 39 (2) (1997) 187–220.
- [4] B. Mourrain, V.Y. Pan, Multivariate polynomials, duality and structured matrices, in: *Proceedings Version in STOC'98*, *Journal of Complexity* 16 (1) (2000) 110–180.
- [5] V.Y. Pan, Some recent algebraic/numerical algorithms, in: *Electronic Proceedings of IMACS/ACA98*, 1998. Available at <http://www-troja.fjfi.cvut.cz/aca98/sessions/approximate>.
- [6] V.Y. Pan, *Structured Matrices and Polynomials: Unified Superfast Algorithms*, Birkhäuser/Springer, Boston/New York, 2001.
- [7] M. Elkadi, B. Mourrain, *Introduction à la résolution des systèmes polynomiaux*, Springer, Berlin, 2007.
- [8] Dongming Wang, Li-Hong Zhi (Eds.), *Symbolic-Numeric Computation*, Birkhäuser, Basel, Boston, 2007.
- [9] J. Verschelde, S. Watt (Eds.), *Proceedings of the 2007 Intern. Workshop on Symbolic-Numeric Computation, SNC2007*, 2007, London, Ontario, Canada, ACM Press, NY, 2007.
- [10] I.S. Kotsireas, K. Shirayanagi (Eds.), *Proceedings of the 2009 Intern. Workshop on Symbolic-Numeric Computation, SNC2009*, 2009, Kyoto, Japan, ACM Press, NY, 2009.
- [11] I.Z. Emiris, B. Mourrain, V.Y. Pan (Eds.), *Special Issue on Algebraic and Numerical Algorithms*, *Theoretical Computer Science* 315 (2–3) (2004) 307–672.
- [12] D.A. Bini, V.Y. Pan, J. Verschelde (Eds.), *Special Issue on Symbolic-Numerical Algorithms*, *Theoretical Computer Science* 409 (2) (2008) 155–157.

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